AAT Level 4

Applied Management Accounting

Chapter 4

Linear Programming

A Comprehensive Study Guide

Assessment Criteria: Linear Programming (LO 3.4)

PART ONE: FOUNDATION KNOWLEDGE

Section 1: Introduction to Linear Programming

1.1 Building on Chapter 3

In Chapter 3, we learned how to handle situations where ONE resource limits production. We used key factor analysis (limiting factor analysis) to decide which products to prioritise.

But what happens when there are MULTIPLE limiting factors? For example:

- Both labour AND materials are in short supply
- Both machine time AND factory space are constrained
- Multiple departments each have limited capacity

When there are multiple constraints, we need a more powerful technique: Linear Programming.

1.2 What is Linear Programming?

Linear programming (LP) is a mathematical technique for finding the optimal solution when there are multiple constraints. It helps organisations:

- Maximise contribution (or profit) given resource limitations
- Minimise costs while meeting minimum requirements

The word "linear" means we assume straight-line relationships - for example, if one unit uses 2 hours, then 10 units use 20 hours (no economies of scale).

1.3 When to Use Which Technique

Situation	Technique to Use
ONE limiting factor	Key Factor Analysis (Chapter 3)
MULTIPLE limiting factors	Linear Programming (Chapter 4)
No limiting factor	Make all products to meet demand

□ Note: In the AAT AMAC assessment, linear programming will only involve TWO products (x and y) and TWO or MORE constraints.

1.4 Two Types of Linear Programming Problems

Maximisation	Minimisation
Goal: Maximise contribution/profit	Goal: Minimise total cost
Most common type in exams	Less common but may appear
Move AWAY from origin	Move TOWARDS origin

Section 2: Assumptions and Limitations

2.1 Key Assumptions

Linear programming is based on several assumptions that may not always hold true in reality:

Assumption 1: Single Quantifiable Objective

We assume there's one clear goal (maximise contribution OR minimise cost).

Reality: Businesses often have multiple objectives - maximise profit while minimising risk, improving quality, maintaining customer satisfaction, etc.

Assumption 2: Constant Resource Usage

Each product always uses the same amount of each resource per unit.

Reality: Learning effects may reduce labour time as workers gain experience. Bulk purchasing may change material quantities.

Assumption 3: Constant Contribution Per Unit

The contribution per unit stays the same regardless of volume.

Reality: Selling prices may need to be reduced to sell more units. Economies of scale may reduce costs at higher volumes.

Assumption 4: Product Independence

Products can be made and sold independently of each other.

Reality: Some products are complementary (razors and blades) or may share marketing costs.

Assumption 5: Short-Term Scenario

Fixed costs are ignored because we're making short-term decisions.

Reality: Long-term decisions would need to consider fixed costs and strategic factors.

□ Important: Understanding these limitations helps you critically evaluate linear programming solutions in practice.

Section 3: The Six-Step Approach - DVOCGS

3.1 Memory Aid: DVOCGS

Remember "DVOCGS" (pronounced "Dee-Vogs"):

Step	Letter	Action
1	**D**	Define the variables (let x = ..., let y = ...)
2	**V**	formulate the objectiVe function (Maximise C = ...)
3	**O**	state the cOnstraints (inequalities)
4	**C**	Chart/graph - draw constraints, identify feasible region
5	**G**	find the optimal point (Graph method or simultaneous equations)
6	**S**	Solve and state the answer

3.2 Step 1: Define the Variables

The variables represent the quantities we're trying to determine - typically how many units of each product to make.

Convention: Use x and y (though any letters work).

Example:

Let x = number of units of Product A manufactured

Let y = number of units of Product B manufactured

□ Exam Tip: Always write out your variable definitions clearly - this prevents confusion and earns method marks.

3.3 Step 2: Formulate the Objective Function

The objective function expresses what we want to maximise or minimise.

```
Maximise C = (contribution per x) × x + (contribution per y) × y
```

Example:

If Product A contributes £5 per unit and Product B contributes £8 per unit:

```
Maximise C = 5x + 8y
```

3.4 Step 3: State the Constraints

Constraints are the limitations we face. Each constraint becomes an inequality.

Types of Constraints:

- Resource constraints: $3x + 2y \leq 100$ (can't use more than available)
- Demand constraints: $x \leq 500$ (can't sell more than demand)
- Minimum requirements: $y \geq 50$ (must make at least this much)
- Non-negativity: $x, y \geq 0$ (can't make negative quantities)

□ Important: Always include the non-negativity constraint ($x, y \geq 0$) - you cannot produce negative units!

3.5 Step 4: Draw the Graph

Each constraint line divides the graph into feasible and infeasible regions.

To plot a constraint line:

1. Convert the inequality to an equation (replace ≤ or ≥ with =)
2. Find two points on the line (easiest: where x=0 and where y=0)
3. Draw the line connecting these points
4. Shade the feasible side (where the inequality is satisfied)

The feasible region is where ALL constraints are satisfied simultaneously.

3.6 Step 5: Find the Optimal Point

There are two methods to find the optimal solution:

Method 1: Iso-contribution Line

- Draw a line representing the objective function for any convenient value
- Move it parallel, away from origin (for maximisation) or towards origin (for minimisation)
- The last point it touches in the feasible region is optimal

Method 2: Simultaneous Equations

- Identify which two constraint lines intersect at the optimal corner
- Solve the equations simultaneously to find exact coordinates
- Calculate contribution at this point

3.7 Step 6: State the Answer

Clearly state:

- The optimal production plan (how many of x and y to make)
- The maximum contribution (or minimum cost) achieved
- Any other information requested in the question

Section 4: Formulating the Problem - Complete Example

The Scenario: Melody Instruments Ltd

Melody Instruments makes two products: Guitars (G) and Keyboards (K).

	Guitar	Keyboard
Assembly time (hours)	4	6
Testing time (hours)	2	3
Contribution per unit	£80	£120
Maximum demand	300 units	200 units

Available hours: Assembly 2,000 hours | Testing 900 hours

Required: Formulate the linear programming problem.

Solution

Step 1: Define the Variables

Let x = number of Guitars manufactured

Let y = number of Keyboards manufactured

Step 2: Formulate the Objective Function

Each Guitar contributes £80, each Keyboard contributes £120.

$$\text{Maximise } C = 80x + 120y$$

Step 3: State the Constraints

Assembly constraint:

Guitars need 4 hours each, Keyboards need 6 hours each. Maximum 2,000 hours.

$$4x + 6y \leq 2{,}000$$

Testing constraint:

Guitars need 2 hours each, Keyboards need 3 hours each. Maximum 900 hours.

$$2x + 3y \leq 900$$

Demand constraints:

$$x \leq 300 \quad \text{(Guitar demand)}$$

$$y \leq 200 \quad \text{(Keyboard demand)}$$

Non-negativity:

$$x, y \geq 0$$

Complete Formulation

$$\text{Maximise } C = 80x + 120y$$

Subject to:

$$4x + 6y \leq 2{,}000 \text{ (Assembly)}$$

$$2x + 3y \leq 900 \text{ (Testing)}$$

$$x \leq 300,\ y \leq 200 \text{ (Demand)}$$

$$x, y \geq 0 \text{ (Non-negativity)}$$

Section 5: Drawing the Graph

5.1 Plotting Constraint Lines

To plot each constraint, we convert the inequality to an equation and find where it crosses the axes.

Assembly constraint: 4x + 6y = 2,000

When x = 0: 6y = 2,000, so y = 333.3

When y = 0: 4x = 2,000, so x = 500

Plot points (0, 333) and (500, 0) and draw a line.

Testing constraint: 2x + 3y = 900

When x = 0: 3y = 900, so y = 300

When y = 0: 2x = 900, so x = 450

Plot points (0, 300) and (450, 0) and draw a line.

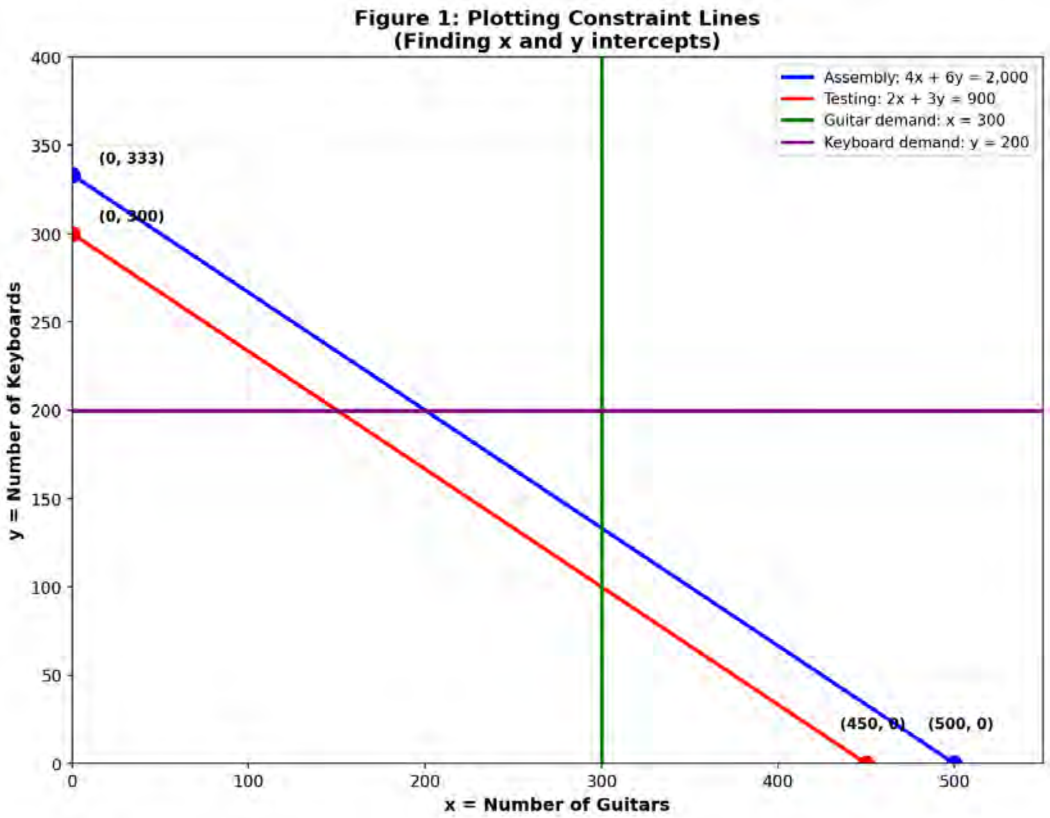

Figure 1: Plotting Constraint Lines (Melody Instruments Example)

Demand constraints:

x = 300 is a vertical line

y = 200 is a horizontal line

5.2 The Feasible Region

The feasible region is the area that satisfies ALL constraints simultaneously.

For "≤" constraints: the feasible area is BELOW and to the LEFT of the line

For "≥" constraints: the feasible area is ABOVE and to the RIGHT of the line

The feasible region for our example is bounded by the axes, the demand lines, and whichever resource constraint is most restrictive at each point.

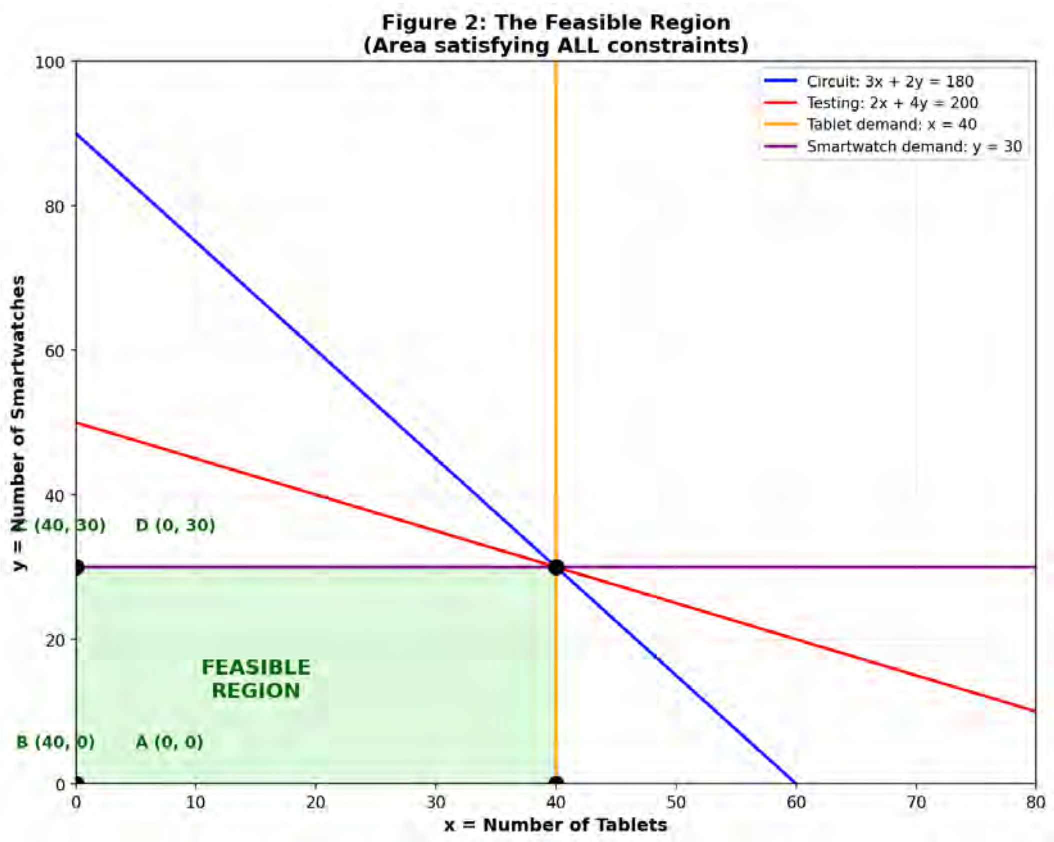

Figure 2: The Feasible Region (Area Satisfying All Constraints)

5.3 Identifying Corner Points (Vertices)

The optimal solution always occurs at a corner point (vertex) of the feasible region.

For our example, the corner points are:

- Point A: (0, 0) - the origin
- Point B: (0, 200) - where y = 200 meets the y-axis
- Point C: where y = 200 meets testing constraint
- Point D: where assembly and testing constraints intersect
- Point E: (300, 0) - where x = 300 meets the x-axis

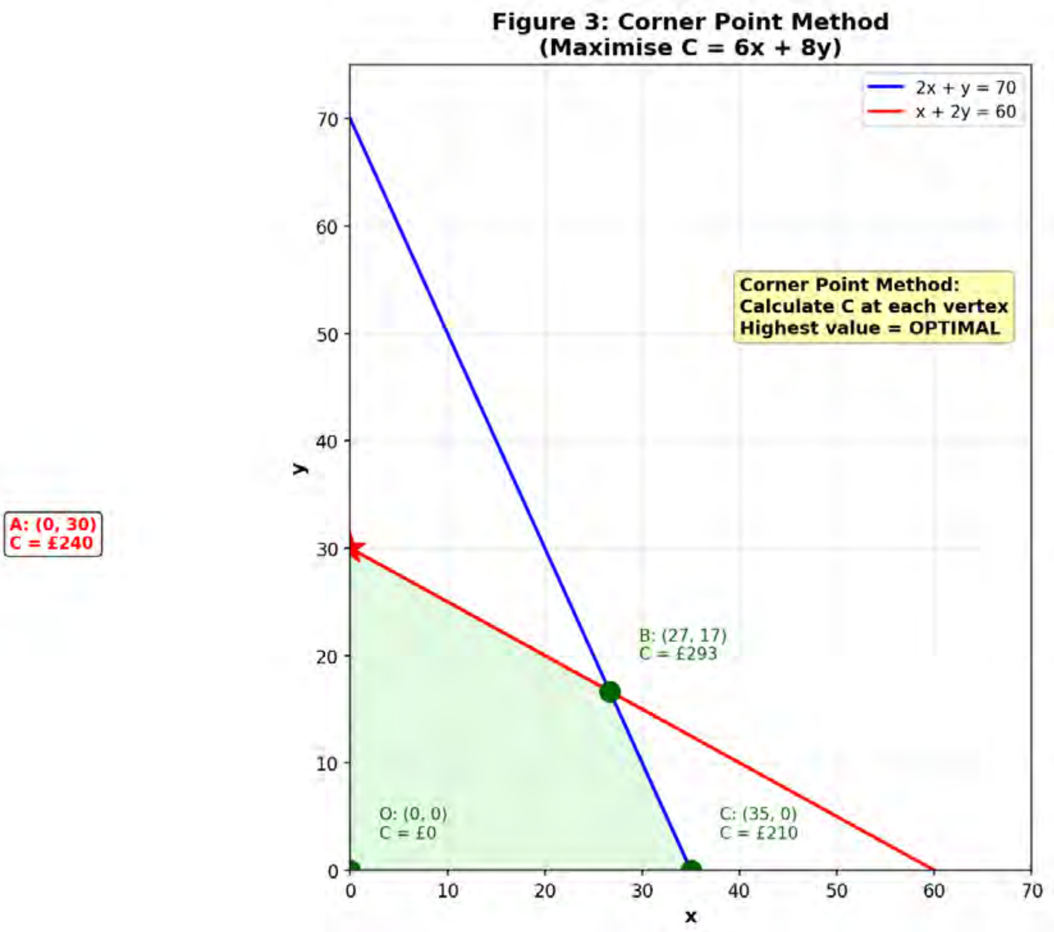

Figure 3: Corner Point Method (Testing Each Vertex)

□ Exam Tip: In the exam, you won't need to draw the graph perfectly, but understanding it helps you identify which constraints are binding at the optimal point.

Section 6: The Iso-Contribution Line Method

6.1 What is an Iso-Contribution Line?

An iso-contribution line shows all combinations of x and y that give the SAME total contribution.

"Iso" means "equal" - so it's a line of equal contribution.

6.2 How to Draw It

Take the objective function and set it equal to any convenient number.

Example: C = 80x + 120y

Let's pick C = 2,400 (a multiple of both 80 and 120 for easy calculation):

80x + 120y = 2,400

When x = 0: y = 20

When y = 0: x = 30

Draw a line from (0, 20) to (30, 0).

6.3 Finding the Optimal Point

For MAXIMISATION:

1. Draw the iso-contribution line
2. Place a ruler along this line
3. Slide the ruler parallel to the line, AWAY from the origin
4. The LAST point the line touches in the feasible region is optimal

For MINIMISATION:

1. Slide the ruler TOWARDS the origin
2. The FIRST point the line touches in the feasible region is optimal

6.4 Why This Works

All iso-contribution lines for the same objective function are parallel (same slope).

Lines further from the origin represent higher contribution values.

We want the highest contribution that's still achievable (within feasible region).

□ Note: You can draw multiple iso-contribution lines to visualise how contribution increases as you move away from the origin.

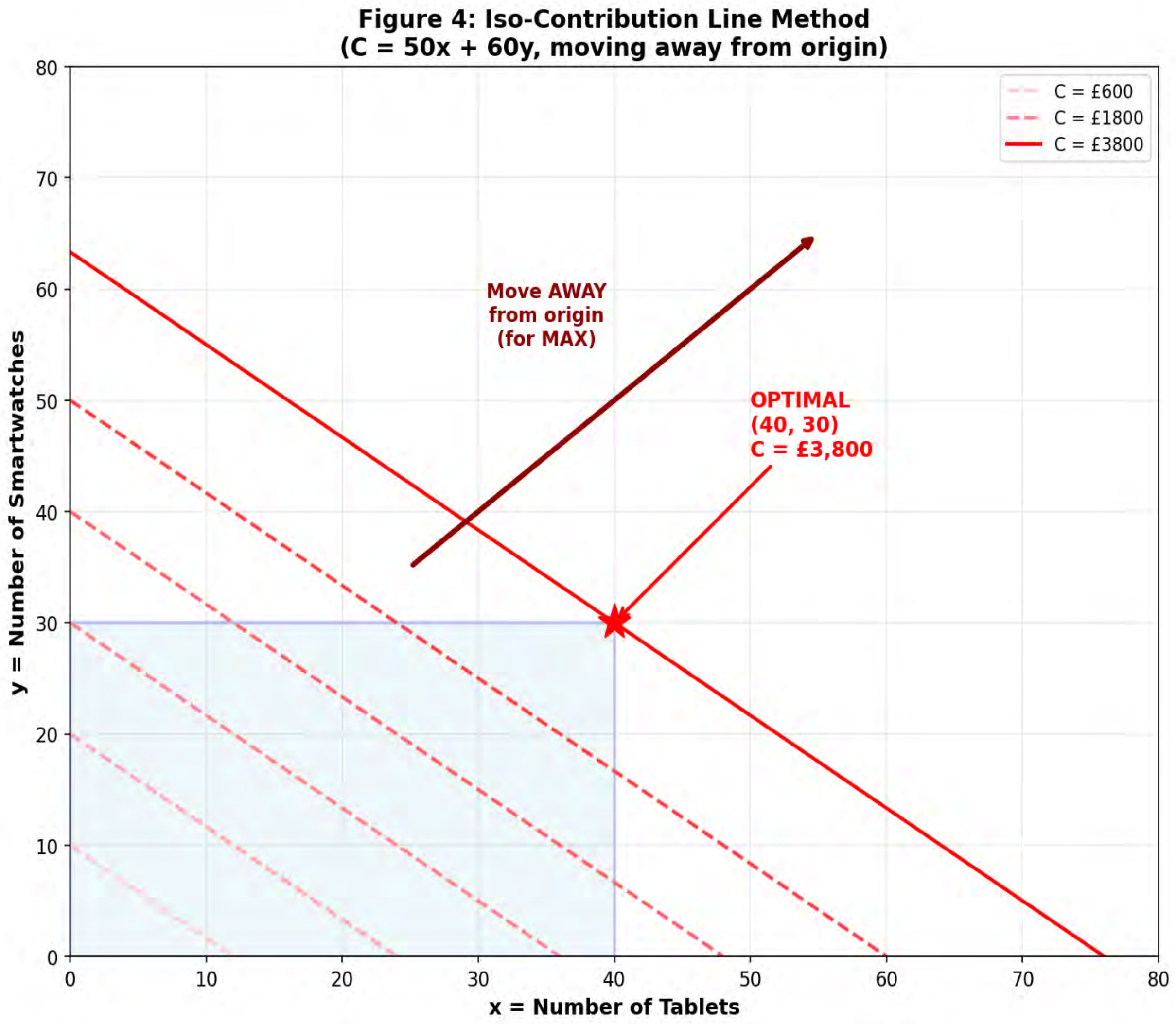

Figure 4: Iso-Contribution Line Method (Moving Away from Origin for Maximisation)

Section 7: Simultaneous Equations Method

7.1 When to Use This Method

Use simultaneous equations when:

- You need exact coordinates of a corner point
- The graph is hard to read precisely
- The exam question asks you to use this method

This method involves solving two equations together to find their intersection point.

7.2 The Elimination Method

The most common approach is to eliminate one variable:

1. Label your equations (i) and (ii)
2. Multiply one or both equations so that one variable has the same coefficient in both
3. Subtract one equation from the other to eliminate that variable
4. Solve for the remaining variable
5. Substitute back to find the other variable
6. Check your answer in the original equations

7.3 Worked Example

Find where these two constraints intersect:

Assembly: $4x + 6y = 2{,}000$ ... (i)

Testing: $2x + 3y = 900$ ... (ii)

Solution

Step 1: Make coefficients match

Multiply equation (ii) by 2:

$4x + 6y = 1{,}800$ ... (iii)

Step 2: Subtract to eliminate

Subtract (iii) from (i):

$(4x + 6y) - (4x + 6y) = 2{,}000 - 1{,}800$

$0 = 200$

Wait - this gives us 0 = 200, which is impossible!

This tells us these two lines are PARALLEL and never intersect.

What This Means

When we get an impossible result, the lines are parallel. Looking at our constraints:

$4x + 6y = 2{,}000$ can be simplified to $2x + 3y = 1{,}000$

$2x + 3y = 900$

These have the same left-hand side but different right-hand sides - parallel lines!

The testing constraint (900) is more restrictive than assembly (1,000), so testing is the binding constraint.

7.4 Another Example - Lines That DO Intersect

Constraint 1: 3x + 4y = 120 ... (i)

Constraint 2: 5x + 2y = 100 ... (ii)

Step 1: Make coefficients match

To eliminate y, multiply (i) by 1 and (ii) by 2:

3x + 4y = 120 ... (i)

10x + 4y = 200 ... (iii)

Step 2: Subtract

(10x + 4y) - (3x + 4y) = 200 - 120

7x = 80

x = 80/7 = 11.43

Step 3: Substitute back

3(11.43) + 4y = 120

34.29 + 4y = 120

4y = 85.71

y = 21.43

Step 4: Check

In equation (ii): 5(11.43) + 2(21.43) = 57.15 + 42.86 = 100 ✓

The intersection point is approximately (11.43, 21.43).

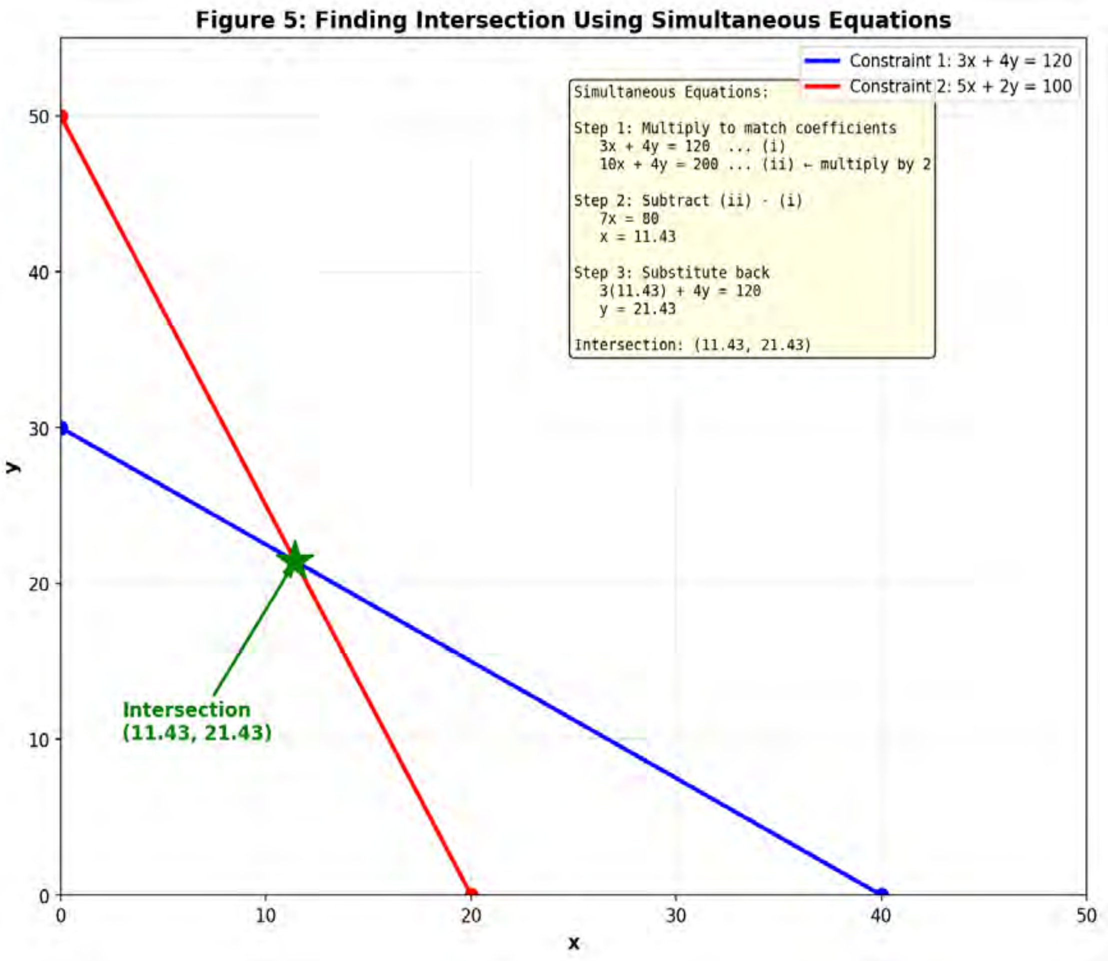

Figure 5: Finding Intersection Using Simultaneous Equations

Section 8: Complete Worked Example

The Scenario: TechGadgets Ltd

TechGadgets produces two products: Tablets (T) and Smartwatches (S).

Resource	Tablet (x)	Smartwatch (y)
Circuit assembly (hours)	3	2
Quality testing (hours)	2	4
Contribution per unit	£50	£60

Available: Circuit assembly 180 hours, Quality testing 200 hours per week.

Maximum demand: Tablets 40 units, Smartwatches 30 units per week.

Required: Find the optimal production plan.

Solution

Step 1: Define Variables

Let x = number of Tablets produced

Let y = number of Smartwatches produced

Step 2: Objective Function

```
Maximise C = 50x + 60y
```

Step 3: Constraints

Circuit assembly: $3x + 2y \leq 180$

Quality testing: $2x + 4y \leq 200$

Tablet demand: $x \leq 40$

Smartwatch demand: $y \leq 30$

Non-negativity: $x, y \geq 0$

Step 4: Plot the Lines

Circuit: 3x + 2y = 180

$x = 0 \rightarrow y = 90$ | $y = 0 \rightarrow x = 60$

Testing: 2x + 4y = 200

$x = 0 \rightarrow y = 50$ | $y = 0 \rightarrow x = 100$

Demand: x = 40, y = 30

Step 5: Find Optimal Point

From the graph (or by checking corners), the optimal point appears to be where the circuit assembly and testing constraints intersect.

Solve simultaneously:

$3x + 2y = 180$ ... (i)

$2x + 4y = 200$ ... (ii)

Multiply (i) by 2: $6x + 4y = 360$ ... (iii)

Subtract (ii) from (iii):

$(6x + 4y) - (2x + 4y) = 360 - 200$

4x = 160

x = 40

Substitute into (i): 3(40) + 2y = 180

120 + 2y = 180

2y = 60

y = 30

The intersection is at (40, 30).

Step 6: Calculate Contribution

C = 50x + 60y = 50(40) + 60(30) = 2,000 + 1,800 = £3,800

Verify Constraints

Circuit: 3(40) + 2(30) = 120 + 60 = 180 $\leqslant 180$ ✓

Testing: 2(40) + 4(30) = 80 + 120 = 200 $\leqslant 200$ ✓

Demand: 40 $\leqslant 40$ ✓ and 30 ≤ 30 ✓

Answer

Optimal production plan: 40 Tablets and 30 Smartwatches per week

Maximum weekly contribution: £3,800

Section 9: Minimisation Problems

9.1 How Minimisation Differs

In minimisation problems, we want to find the lowest cost solution that still meets all requirements.

Maximisation	Minimisation
Objective: Maximise C = ...	Objective: Minimise C = ...
Resource constraints are ≤	Requirement constraints are ≥
Move iso-line AWAY from origin	Move iso-line TOWARDS origin
Find furthest feasible corner	Find closest feasible corner

9.2 Example Setup

A company must produce at least 100 units total using two processes:

Process A: £5 per unit, produces 2 units per hour

Process B: £8 per unit, produces 3 units per hour

Minimum 20 hours must be worked. Minimise total cost.

Formulation

Let x = hours using Process A

Let y = hours using Process B

Minimise C = 5(2x) + 8(3y) = 10x + 24y

Subject to:

2x + 3y ≥ 100 (minimum output)

x + y ≥ 20 (minimum hours)

x, y ≥ 0

9.3 Graphical Solution

For "≥" constraints, the feasible region is ABOVE and to the RIGHT of each line.

The feasible region is unbounded in the direction away from the origin.

To minimise, we draw the iso-cost line and move it TOWARDS the origin.

The optimal point is the first feasible corner the iso-cost line touches.

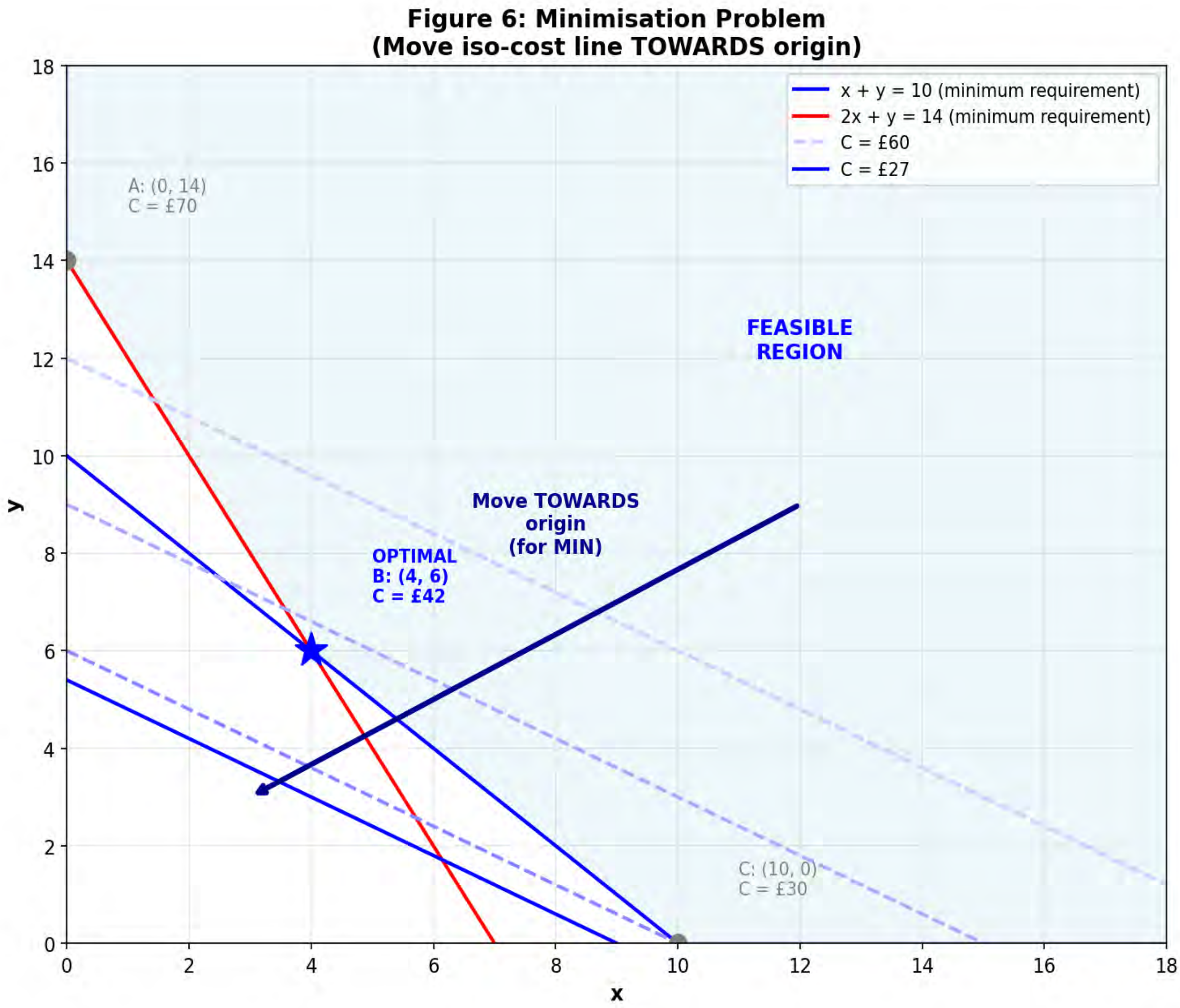

Figure 6: Minimisation Problem (Move Iso-Cost Line Towards Origin)

Section 10: Key Formulas Summary

Formula	Purpose
$C = ax + by$	Objective function
$ax + by \leq c$	Resource constraint (maximisation)
$ax + by \geq c$	Requirement constraint (minimisation)
$x, y \geq 0$	Non-negativity constraint
When $x = 0$, $y = c/b$	Find y-intercept of line
When $y = 0$, $x = c/a$	Find x-intercept of line

PART TWO: PRACTICE AND ASSESSMENT

Section 11: Comprehensive Worked Example - GreenJuice Co

The Scenario

GreenJuice Co produces two smoothie blends: "Berry Blast" (B) and "Tropical Mix" (T).

Ingredient (per batch)	Berry Blast	Tropical Mix
Strawberries (kg)	4	1
Bananas (kg)	2	3
Mangoes (kg)	1	4
Contribution per batch	£25	£30

Weekly availability: Strawberries 80kg, Bananas 90kg, Mangoes 100kg.

Required: Determine the optimal production plan to maximise contribution.

Solution

Step 1: Define Variables

Let x = number of batches of Berry Blast

Let y = number of batches of Tropical Mix

Step 2: Objective Function

$$\text{Maximise } C = 25x + 30y$$

Step 3: State Constraints

Strawberries: $4x + y \leq 80$

Bananas: $2x + 3y \leq 90$

Mangoes: $x + 4y \leq 100$

Non-negativity: $x, y \geq 0$

Step 4: Plot Constraint Lines

Constraint	x-intercept (y=0)	y-intercept (x=0)
4x + y = 80	x = 20	y = 80
2x + 3y = 90	x = 45	y = 30
x + 4y = 100	x = 100	y = 25

Step 5: Find Corner Points

The feasible region has several corner points. We need to find the optimal one.

From visual inspection, the optimal point appears to be where strawberry and banana constraints intersect.

Solve: $4x + y = 80$ and $2x + 3y = 90$

From equation 1: $y = 80 - 4x$

Substitute into equation 2: $2x + 3(80 - 4x) = 90$

$2x + 240 - 12x = 90$

-10x = -150

x = 15

y = 80 - 4(15) = 80 - 60 = 20

Intersection point: (15, 20)

Step 6: Verify and Calculate

Check constraints:

Strawberries: 4(15) + 20 = 80 $\leqslant 80$ ✓

Bananas: 2(15) + 3(20) = 30 + 60 = 90 $\leqslant 90$ ✓

Mangoes: 15 + 4(20) = 15 + 80 = 95 $\leqslant 100$ ✓

Calculate contribution:

C = 25(15) + 30(20) = 375 + 600 = £975

Answer

Optimal production: 15 batches of Berry Blast, 20 batches of Tropical Mix

Maximum weekly contribution: £975

Section 12: Practice Questions

Question 1: Problem Formulation (10 marks)

Craft Pottery makes two products: Vases (V) and Bowls (B).

	Vase	Bowl
Clay per unit (kg)	3	2
Kiln time (hours)	2	1
Contribution per unit	£15	£10

Weekly resources: Clay 120 kg, Kiln time 70 hours. Max demand: Vases 30, Bowls 40.

Required:

(a) Define the variables. (2 marks)

(b) State the objective function. (2 marks)

(c) State all constraints including non-negativity. (6 marks)

Question 2: Simultaneous Equations (10 marks)

Solve the following pairs of equations simultaneously:

(a) $3x + 2y = 24$ and $x + 4y = 22$ (5 marks)

(b) $5x + 3y = 39$ and $2x + 5y = 37$ (5 marks)

Question 3: Full Linear Programming Problem (25 marks)

FitGear Ltd produces two exercise products: Dumbbells (D) and Kettlebells (K).

	Dumbbell	Kettlebell
Casting time (hours)	2	3
Finishing time (hours)	4	2
Contribution per unit	£12	£18

Available: Casting 60 hours, Finishing 80 hours per week.

Demand: Dumbbells max 25 units, Kettlebells max 15 units.

Required:

(a) Formulate the linear programming problem. (8 marks)

(b) Find the coordinates of the corner points. (9 marks)

(c) Determine the optimal production plan. (4 marks)

(d) Calculate the maximum contribution. (4 marks)

Question 4: Graphical Interpretation (10 marks)

A company makes products X and Y with objective function: Maximise $C = 6x + 8y$

The constraints have created a feasible region with corners at: (0,0), (0,30), (20,20), (30,10), (35,0)

Required:

(a) Calculate the contribution at each corner point. (5 marks)

(b) Identify the optimal production plan. (3 marks)

(c) State the maximum contribution. (2 marks)

Question 5: Multiple Choice Questions

5.1 Linear programming should be used when:

A) There is one limiting factor

B) There are no limiting factors

C) There are multiple limiting factors

D) Fixed costs need to be allocated

5.2 The feasible region represents:

A) All points where contribution is maximised

B) All points that satisfy all constraints

C) All points on the objective function line

D) All points where x and y are negative

5.3 The optimal solution in linear programming:

A) Always occurs at the origin

B) Always occurs at a corner of the feasible region

C) Occurs anywhere in the feasible region

D) Occurs outside the feasible region

5.4 If $3x + 5y = 30$, when $x = 0$, y equals:

A) 30

B) 10

C) 6

D) 5

5.5 The iso-contribution line:

A) Shows where constraints intersect

B) Shows all combinations giving equal contribution

C) Marks the boundary of the feasible region

D) Always passes through the origin

Section 13: Answers to Practice Questions

Answer 1: Problem Formulation

(a) Variables

Let x = number of Vases produced

Let y = number of Bowls produced

(b) Objective function

```
Maximise C = 15x + 10y
```

(c) Constraints

Clay: $3x + 2y \le 120$

Kiln: $2x + y \le 70$

Vase demand: $x \le 30$

Bowl demand: $y \le 40$

Non-negativity: $x, y \ge 0$

Answer 2: Simultaneous Equations

(a) 3x + 2y = 24 and x + 4y = 22

Multiply equation 2 by 3: $3x + 12y = 66$

Subtract equation 1: $(3x + 12y) - (3x + 2y) = 66 - 24$

$10y = 42$, so $y = 4.2$

Substitute: $x + 4(4.2) = 22$

$x = 22 - 16.8 = 5.2$

Solution: x = 5.2, y = 4.2

(b) 5x + 3y = 39 and 2x + 5y = 37

Multiply equation 1 by 5: $25x + 15y = 195$

Multiply equation 2 by 3: $6x + 15y = 111$

Subtract: $19x = 84$, so $x = 4.42$

Substitute: $5(4.42) + 3y = 39$

$22.1 + 3y = 39$, $3y = 16.9$, $y = 5.63$

Solution: x = 4.42, y = 5.63

Answer 3: Full LP Problem

(a) Formulation

Let x = number of Dumbbells, y = number of Kettlebells

Maximise $C = 12x + 18y$

Subject to:

$2x + 3y \le 60$ (Casting)

$4x + 2y \le 80$ (Finishing)

$x \le 25$, $y \le 15$ (Demand)

x, y ≥ 0

(b) Corner points

Origin: (0, 0)

y-axis at y=15: 2(0) + 3(15) = 45 < 60 ✓, so (0, 15)

Casting meets y=15: Not applicable (casting allows y=20 when x=0)

Casting meets finishing: 2x + 3y = 60 and 4x + 2y = 80

Multiply casting by 2: 4x + 6y = 120

Subtract: 4y = 40, y = 10

Substitute: 2x + 30 = 60, x = 15

Intersection: (15, 10)

x-axis at x=20: 4(20) + 2(0) = 80 ≤ 80 ✓, but check casting: 2(20) = 40 < 60 ✓

So (20, 0)

(c) Optimal production

Calculate contribution at each corner:

(0, 0): C = 0

(0, 15): C = 18(15) = £270

(15, 10): C = 12(15) + 18(10) = 180 + 180 = £360

(20, 0): C = 12(20) = £240

Optimal: 15 Dumbbells and 10 Kettlebells

(d) Maximum contribution

Maximum contribution = £360 per week

Answer 4: Graphical Interpretation

(a) Contribution at each corner

Corner	Calculation	Contribution
(0, 0)	6(0) + 8(0)	£0
(0, 30)	6(0) + 8(30)	£240
(20, 20)	6(20) + 8(20)	£280
(30, 10)	6(30) + 8(10)	£260
(35, 0)	6(35) + 8(0)	£210

(b) & (c) Optimal solution

Optimal: x = 20, y = 20 | Maximum contribution = £280

Answer 5: Multiple Choice

Question	Answer	Explanation
5.1	C	LP is used when there are multiple limiting factors
5.2	B	Feasible region = area satisfying ALL constraints
5.3	B	Optimal solution always at a corner/vertex
5.4	C	3(0) + 5y = 30, so y = 6
5.5	B	Iso = equal, shows points of equal contribution

PART THREE: SUMMARY AND EXAM PREPARATION

Section 14: Key Points Summary

When to Use Linear Programming

ONE limiting factor → Key Factor Analysis (Chapter 3)

MULTIPLE limiting factors → Linear Programming (Chapter 4)

The Six Steps - DVOCGS

Step	Letter	Action
1	**D**	Define variables (let x = ..., let y = ...)
2	**V**	formulate objectiVe function (Max/Min C = ax + by)
3	**O**	state cOnstraints (don't forget non-negativity!)
4	**C**	Chart/graph - plot lines, identify feasible region
5	**G**	find optimal point (Graph/simultaneous equations)
6	**S**	Solve and state the answer clearly

Section 15: Memory Aids and Quick Reference

Finding Line Intercepts

```
For line ax + by = c:

x-intercept (where y = 0): x = c ÷ a

y-intercept (where x = 0): y = c ÷ b
```

Constraint Direction Memory Aid

Symbol	Feasible Side	Problem Type
≤	Below/left of line	Maximisation (resource limit)
≥	Above/right of line	Minimisation (min requirement)

Simultaneous Equations Quick Method

Remember: "Match, Subtract, Substitute"

1. MATCH: Multiply equations to get same coefficient for one variable
2. SUBTRACT: Subtract equations to eliminate that variable
3. SUBSTITUTE: Put the value back into original equation

Iso-Contribution Line Direction

Problem Type	Move Iso-Line
Maximisation	AWAY from origin (further = more)
Minimisation	TOWARDS origin (closer = less)

Section 16: Common Exam Mistakes to Avoid

Mistake 1: Forgetting Non-Negativity Constraint

□ **Wrong:** Stating only resource constraints

✓ **Correct:** Always include x, y ≥ 0

Mistake 2: Wrong Constraint Direction

□ **Wrong:** Using ≥ for resource constraints in maximisation

✓ **Correct:** Resources are limits (≤), requirements are minimums (≥)

Mistake 3: Errors in Simultaneous Equations

□ **Wrong:** Adding instead of subtracting when eliminating

✓ **Correct:** Always verify your answer by substituting back

Mistake 4: Not Checking All Corners

□ **Wrong:** Assuming the first corner found is optimal

✓ **Correct:** Calculate contribution at all corners to find maximum

Mistake 5: Plotting Errors

□ **Wrong:** Confusing x-intercept and y-intercept

✓ **Correct:** x-intercept: where line crosses x-axis (y = 0)

Mistake 6: Wrong Problem Type

□ **Wrong:** Moving iso-line towards origin for maximisation

✓ **Correct:** Max = away from origin, Min = towards origin

Section 17: Exam Tips

Before the Exam

- Memorise DVOCGS - the six-step approach
- Practice simultaneous equations until they become automatic
- Know how to find intercepts quickly (divide c by coefficient)
- Understand both maximisation and minimisation

During the Exam

- Read carefully - is it maximisation or minimisation?
- Write out variable definitions first (earns marks)
- Don't forget non-negativity constraint (x, y ≥ 0)
- Show all workings for simultaneous equations
- Always verify your answer satisfies ALL constraints
- State your final answer clearly - units and contribution

Key Question Triggers

Question Says	What to Do
"Formulate the problem"	Steps 1-3 (variables, objective, constraints)
"Using simultaneous equations"	Use elimination method to solve
"Optimal production plan"	Find coordinates of optimal point
"Maximum contribution"	Substitute optimal point into objective function
"Identify the feasible region"	Describe which constraints bound it

Section 18: Quick Revision Checklist

Knowledge

- ☐ Know when to use LP vs key factor analysis
- ☐ Understand the assumptions/limitations of LP
- ☐ Know the six-step approach (DVOCGS)
- ☐ Understand maximisation vs minimisation

Skills

- ☐ Define variables correctly
- ☐ Write objective functions
- ☐ Formulate constraints (including non-negativity)
- ☐ Find line intercepts
- ☐ Solve simultaneous equations
- ☐ Calculate contribution at corner points

Application

- ☐ Complete a full LP problem from start to finish
- ☐ Verify solutions satisfy all constraints
- ☐ Interpret results and state optimal plan

Section 19: Final Summary

The Core Process

1. DEFINE: Let x = Product A, Let y = Product B
2. OBJECTIVE: Maximise (or Minimise) C = ax + by
3. CONSTRAINTS: Resource $\leq$ Available (plus x, y $\geq$ 0)
4. GRAPH: Plot lines using intercepts, shade feasible region
5. SOLVE: Use iso-line method OR simultaneous equations
6. ANSWER: State optimal plan and maximum contribution

Key Relationships

The optimal solution is ALWAYS at a CORNER of the feasible region

Good luck with your AAT Level 4 exam!

Linear Programming = Multiple constraints optimised mathematically

— End of Chapter 4 Study Guide —